MULTIRESOLUTION FINITE ELEMENT AND BOUNDARY ELEMENT METHODS BASED ON WAVELET AND SUBDIVISION THEORY

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We describe a framework for multiscale finite and boundary element analysis derived from recent ideas in wavelet and subdivision theory. Our wavelet approach is a natural extension to hierarchical finite elements[4], which use primitive wavelets with no vanishing moments. A distinguishing feature of our method is that it allows the construction of second-generation[3] multiwavelets of desired order and smoothness over non-uniformly discretized domains, starting from traditional finite elements. For example, by applying the concepts of finite element subdivision[2] and lifting[3] we can construct a family of Hermite interpolating multiwavelets which (in the simplest case) satisfy the following refinement and wavelet relations[1]:

$$\begin{aligned}
\{\ \varphi^u_{j,k}(x) \quad \varphi^\theta_{j,k}(x)\ \} &= \{\ \varphi^u_{j+1,k}(x) \quad \varphi^\theta_{j+1,k}(x)\ \} \quad + \quad \textstyle\sum_{m\in n(j,k)} \{\ \varphi^u_{j+1,m}(x) \quad \varphi^\theta_{j+1,m}(x)\ \}\mathbf{H}_j^T[k,m] \\
\{\ w^u_{j,m}(x) \quad w^\theta_{j,m}(x)\ \} &= \{\ \varphi^u_{j+1,m}(x) \quad \varphi^\theta_{j+1,m}(x)\ \} \quad - \quad \textstyle\sum_{k\in A(j,m)} \{\ \varphi^u_{j,k}(x) \quad \varphi^\theta_{j,k}(x)\ \}\mathbf{S}_j[k,m]
\end{aligned}$$

Similar equations may be derived in higher dimensions. For example, Figure 1 shows two of the scaling function and wavelet pairs that generalize the Bogner-Fox-Schmidt plate bending elements. Our approach therefore establishes an important connection between displacement-based finite element analysis and multiscale signal processing.

The finite element wavelets that we describe offer several advantages over the "single level" formulation for preconditioning, adaptivity, fast solution of eigenvalue problems and level-of-detail analysis of the solution. Moreover, by an appropriate choice of lifting coefficients, $\mathbf{S}_j[k,m]$, the resulting wavelets may be adapted both to the geometry as well as the differential operator. We present several applications to illustrate our approach.

References

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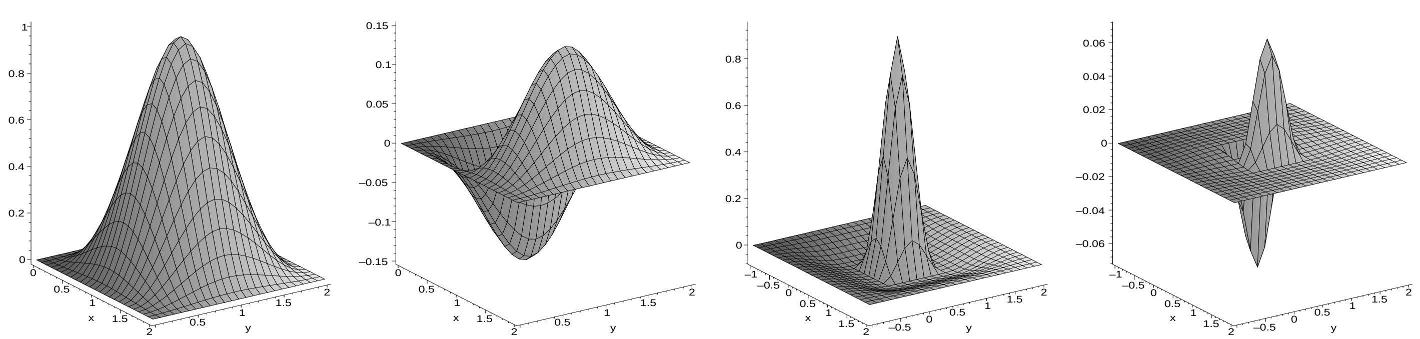

Figure 1: Displacement and rotation scaling functions and wavelets with with four vanishing moments.